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4.2 Weitere Beispiele: $\int_0^b f(x)\,dx$ berechnen.

a) $f(x)=2\cdot x+3$ b) $f(x)=\frac{1}{5}x^2$ c) $f(x)=x^3$ d) $f(x)=3x^2-5x$

zu a):

$$O_n=\frac{b}{n}\cdot f\left(\frac{1\cdot b}{n}\right)+\frac{b}{n}\cdot f\left(\frac{2\cdot b}{n}\right)+\ldots+\frac{b}{n}\cdot f\left(\frac{n\cdot b}{n}\right)$$
$$=\frac{b}{n}\cdot\left(2\cdot\frac{1b}{n}+3\right)+\frac{b}{n}\cdot\left(2\cdot\frac{2b}{n}+3\right)+\ldots+\frac{b}{n}\cdot\left(2\cdot\frac{n\cdot b}{n}+3\right)$$
$$=2\frac{b^2}{n^2}\cdot 1+\frac{b}{n}\cdot 3+2\frac{b^2}{n^2}\cdot 2+\frac{b}{n}\cdot 3+\ldots+2\frac{b^2}{n^2}\cdot n+\frac{b}{n}\cdot 3$$
$$=2\cdot\frac{b^2}{n^2}\cdot[1+2+\ldots+n]+n\cdot\frac{b}{n}\cdot 3 \quad\rightarrow \text{Summenformel anwenden}$$
$$=2\cdot\frac{b^2}{n^2}\cdot\left[\frac{1}{2}\cdot n\cdot(n+1)\right]+b\cdot 3$$
$$=2\cdot\frac{b^2}{n^2}\cdot\left[\frac{1}{2}n^2+\frac{1}{2}n\right]+3b$$
$$=2\cdot\frac{b^2}{n^2}\cdot\frac{1}{2}n^2+2\cdot\frac{b^2}{n^2}\cdot\frac{1}{2}n+3b$$
$$=b^2+\frac{b^2}{n}+3b$$
$$\Rightarrow\int_0^b(2\cdot x+3)\,dx=\lim_{n\to\infty}\left(b^2+\frac{b^2}{n}+3b\right)=\underline{\underline{b^2+3b}}$$

zu b):

$$O_n=\frac{b}{n}\cdot f\left(\frac{1\cdot b}{n}\right)+\frac{b}{n}\cdot f\left(\frac{2\cdot b}{n}\right)+\ldots+\frac{b}{n}\cdot f\left(\frac{n\cdot b}{n}\right)$$
$$=\frac{b}{n}\cdot\frac{1}{5}\cdot\left(\frac{1\cdot b}{n}\right)^2+\frac{b}{n}\cdot\frac{1}{5}\cdot\left(\frac{2\cdot b}{n}\right)^2+\ldots+\frac{b}{n}\cdot\frac{1}{5}\left(\frac{n\cdot b}{n}\right)^2$$
$$=\frac{1}{5}\cdot\frac{b^3}{n^3}\cdot 1^2+\frac{1}{5}\cdot\frac{b^3}{n^3}\cdot 2^2+\ldots+\frac{1}{5}\cdot\frac{b^3}{n^3}\cdot n^2$$
$$=\frac{1}{5}\cdot\frac{b^3}{n^3}\cdot\left[1^2+2^2+\ldots+n^2\right] \quad\rightarrow \text{Summenformel anwenden}$$
$$=\frac{1}{5}\cdot\frac{b^3}{n^3}\cdot\left[\frac{1}{6}\cdot n\cdot(n+1)\cdot(2n+1)\right]$$
$$=\frac{1}{5}\cdot\frac{b^3}{n^3}\cdot\left[\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n\right]$$
$$=\frac{1}{15}\cdot b^3+\frac{1}{10}\cdot\frac{b^3}{n}+\frac{1}{30}\cdot\frac{b^3}{n^2}$$
$$\int_0^b\frac{1}{5}x^2\,dx=\lim_{n\to\infty}\left(\frac{1}{15}\cdot b^3+\frac{1}{10}\cdot\frac{b^3}{n}+\frac{1}{30}\cdot\frac{b^3}{n^2}\right)=\underline{\underline{\frac{1}{15}b^3}}$$

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zu c):

$$O_n=\frac{b}{n}\cdot f\left(\frac{1\cdot b}{n}\right)+\frac{b}{n}\cdot f\left(\frac{2\cdot b}{n}\right)+\ldots+\frac{b}{n}\cdot f\left(\frac{n\cdot b}{n}\right)$$
$$=\frac{b}{n}\cdot\left(\frac{1\cdot b}{n}\right)^3+\frac{b}{n}\cdot\left(\frac{2\cdot b}{n}\right)^3+\ldots+\frac{b}{n}\cdot\left(\frac{n\cdot b}{n}\right)^3$$
$$=\frac{b}{n}\cdot\frac{b^3}{n^3}\cdot 1^3+\frac{b}{n}\cdot\frac{b^3}{n^3}\cdot 2^3+\ldots+\frac{b}{n}\cdot\frac{b^3}{n^3}\cdot n^3$$
$$=\frac{b^4}{n^4}\cdot\left[1^3+2^3+\ldots+n^3\right] \quad\rightarrow \text{Summenformel anwenden}$$
$$=\frac{b^4}{n^4}\cdot\left[\left(\frac{1}{2}\cdot n\cdot(n+1)\right)^2\right]$$
$$=\frac{b^4}{n^4}\cdot\left[\frac{1}{4}\cdot n^4+\frac{1}{4}\cdot n^3\right]$$
$$=\frac{1}{4}b^4+\frac{1}{4}\frac{b^3}{n}$$
$$\Rightarrow\int_0^b x^3\,dx=\lim_{n\to\infty}\left(\frac{1}{4}b^4+\frac{1}{4}\frac{b^3}{n}\right)=\underline{\underline{\frac{1}{4}b^4}}$$

zu d):

$$O_n=\frac{b}{n}\cdot f\left(\frac{1\cdot b}{n}\right)+\frac{b}{n}\cdot f\left(\frac{2\cdot b}{n}\right)+\ldots+\frac{b}{n}\cdot f\left(\frac{n\cdot b}{n}\right)$$
$$=\frac{b}{n}\cdot\left(3\cdot\left(\frac{1\cdot b}{n}\right)^2-5\cdot\left(\frac{1\cdot b}{n}\right)\right)+\frac{b}{n}\cdot\left(3\cdot\left(\frac{2\cdot b}{n}\right)^2-5\cdot\left(\frac{2\cdot b}{n}\right)\right)+\ldots+\frac{b}{n}\cdot\left(3\cdot\left(\frac{n\cdot b}{n}\right)^2-5\cdot\left(\frac{n\cdot b}{n}\right)\right)$$
$$=\frac{b}{n}\cdot 3\cdot\frac{1^2b^2}{n^2}-\frac{b}{n}\cdot 5\cdot\frac{1\cdot b}{n}+\frac{b}{n}\cdot 3\cdot\frac{2^2b^2}{n^2}-\frac{b}{n}\cdot 5\cdot\frac{2\cdot b}{n}+\ldots+\frac{b}{n}\cdot 3\cdot\frac{n^2b^2}{n^2}-5\cdot\frac{b}{n}\cdot\frac{n\cdot b}{n}$$
$$=3\cdot\frac{b^3}{n^3}\cdot 1^2-5\cdot\frac{b^2}{n^2}\cdot 1+3\frac{b^3}{n^3}\cdot 2^2-5\frac{b^2}{n^2}\cdot 2+\ldots+3\frac{b^3}{n^3}\cdot n^2-5\frac{b^2}{n^2}\cdot n$$
$$=3\frac{b^3}{n^3}\cdot\left[1^2+2^2+\ldots+n^2\right]-5\cdot\frac{b^2}{n^2}\cdot[1+2+\ldots+n] \quad\rightarrow \text{Summenformel!}$$
$$=3\cdot\frac{b^3}{n^3}\left[\frac{1}{6}\cdot n\cdot(n+1)\cdot(2n+1)\right]-5\cdot\frac{b^2}{n^2}\cdot\left[\frac{1}{2}\cdot n\cdot(n+1)\right]$$
$$=3\cdot\frac{b^3}{n^3}\cdot\left[\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n\right]-5\frac{b^2}{n^2}\cdot\left[\frac{1}{2}n^2+\frac{1}{2}n\right]$$
$$=b^3+\frac{3}{2}\frac{b^3}{n}+\frac{1}{2}\frac{b^3}{n^2}-\frac{5}{2}b^2-\frac{5}{2}\frac{b^2}{n}$$
$$\int_0^b(3x^2-5x)\,dx=\lim_{n\to\infty}O_n=\underline{\underline{b^3-\frac{5}{2}b^2}}$$